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STUDY OF EVOLUTION OF LINEARIZED DISTURBANCES IN A STRATIFIED BOUNDED COUETTE FLOW

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ABSTRACT

Using initial value problem approach the evolution of linearized disturbances in a stratified shear flow is studied. The resulting equation in time posed by using Fourier transform is solved for the Fourier amplitudes for the case of bounded couette flow with point source of the field of transverse velocity and density as the initial distributions. For small values of Brunt Väisälä frequency the perturbation solutions are obtained.

Keywords: Stratified bounded couette flow, initial value problem, fourier transform, Brunt Väisälä frequency.

I. INTRODUCTION

The stability of stratified shear flow is of importance in atmospheric and oceanographic environments and has been investigated by many researchers. By using initial-value problem approach, Eliassen, Hoiland and Riis (1953) they showed that a disturbance originating from arbitrary initial conditions in a flow between two parallel walls would behave asymptotically. Miles (1961) established the conjecture that a sufficient condition for stability in a parallel stratified, inviscid flow is that the local Richardson number J_0 should every where exceed $\frac{1}{4}$. Kuo (1963) found that the plane Couette flow in stably and unstably stratified fluids to be more unstable when it is bounded both above and below than when its depth is infinite. Chimonas (1979) studied the stability of stratified shear flow and concluded that the flow will be unstable if the local Richardson number falls below $\frac{1}{4}$ anywhere in the flow. Brown and Stewartson (1980) have resolved the controversy surrounding the decay rate in favour of original results of Eliassen *et al* (1953).

In this paper, we have extended the work of Criminale and Drazin (1990), for the case of stratified bounded coquette flow. The essence of the approach is as follows: Taking a multilayered basic flow with piecewise linear velocity profile, complete general solutions to the linearized equations of motion are obtained as functions of all space variables and time, when posed as initial-value problems. The distributions are resolved into two components, rotational and irrotational. The solution for the hypothetical initial-value problem for which the basic flow is unbounded but coincides with the actual flow in the layer is the rotational solution. The irrotational solution in each layer is specified uniquely by satisfying the interfacial conditions and boundary conditions at infinity. Vijayalakshmi and Balagondar, (2017) studied the evolution of linearized perturbations in a magnetohydrodynamic baroclinic couette flow using initial value problem approach.

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II. MATHEMATICAL FORMULATION

We consider an inviscid, incompressible, inviscid fluid of density ρ moving with velocity $\vec{q}$ under the influence of gravity $\vec{g}$ directed in the negative y-direction. We assume that the fluid is Boussinesq for which motion is governed by the equations

$$\nabla.\vec{q} = 0, \tag{2.1}$$

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right) = -\nabla p + \rho\vec{g}, \tag{2.2}$$

$$\frac{\partial \rho}{\partial t} + (\vec{q}.\nabla)\rho = 0, \tag{2.3}$$

where p is the pressure.

For linear stability analysis, we superimpose a small perturbation upon the mean flow i,e.,

$$\vec{q} = \vec{q}_0 + \vec{q}',\ p = p_0(y) + p',\ \rho = \rho_0(y) + \rho' \tag{2.4}$$

where

$$\vec{q}_0 = (U(y) = \sigma y, 0, 0),\ p = p_0(y),\ \rho = \rho_0(y) \tag{2.5}$$

are the basic unperturbed equilibrium velocity, pressure and density and $\vec{q}'$, p' and ρ' are the perturbed quantities of velocity, pressure and density respectively. σ is the shear intensity which is a constant

To study the evolution of linearized disturbances in a stratified shear flow, we linearize equations (2.1)–(2.3) using (2.4), the linearized differential equations of motion (neglecting the primes) by (i) defining the transformation of co-ordinates of the form

$$T = t,\ \xi = x - \sigma y t,\ \eta = y,\ \zeta = z, \tag{2.6}$$

(ii) employing three - dimensional Fourier transformation of the form

$$\hat{u}(\alpha;\beta;\gamma;T) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(\xi;\eta;\zeta;T)\, e^{i(\alpha\xi + \beta\eta + \gamma\zeta)}\, d\xi\, d\eta\, d\zeta, \tag{2.7}$$

and with similar expressions for $\hat{v}$, $\hat{w}$, $\hat{p}$ and $\hat{\rho}$,
and (iii) making use of Squire transformation

$$\bar{u} = \frac{\alpha\hat{u} + \gamma\hat{w}}{\bar{\alpha}},\ \bar{w} = \frac{-\gamma\hat{u} + \alpha\hat{w}}{\bar{\alpha}} \tag{2.8}$$

which are the velocity components in the $\bar{\alpha}$ and φ directions.

Eliminating $\hat{p}$, the set of equations (2.1)–(2.3) is reduced to

$$\frac{d^2}{dT^2}\left(K^2\hat{v}\right) + N^2\bar{\alpha}^2\hat{v} = 0, \tag{2.9}$$

where $K^2 = \bar{\alpha}^2 + (\beta - \sigma\alpha T)^2$ and $\bar{\alpha}^2 = (\alpha^2 + \gamma^2)$.

$N^2 = -\frac{g}{\rho_0}\frac{d\rho_0}{dy}$, ρ_0 is the equilibrium density, N is the Brunt Väisälä frequency.

Equation (2.9) is solved with appropriate initial conditions for $\hat{v}$ and $\hat{\rho}$, the other velocity components $\hat{u}$, $\hat{w}$ can be obtained by inverting the relations The pressure $\hat{p}$ is obtained by taking the divergence of

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the momentum equations. and it is found that $\hat{p} = \dfrac{-i\left(2\sigma\alpha\rho_0\hat{v} - g(\beta-\sigma\alpha T)\hat{\rho}\right)}{K^2}$, for $K^2 \neq 0$

Two sets of solutions exist for equation (2.9), when $K^2 \neq 0$, the disturbance is rotational and for $K^2 = 0$, the disturbance is irrotational. The vanishing of the product $K^2\hat{v}$ corresponds to Laplace equation $\nabla^2\hat{v} = 0$ in real space. We denote $\hat{v}$ as $\hat{v}_R$ when $K^2 \neq 0$ which is called the rotational solution and $\hat{v}$ as $\hat{v}_I$ when $K^2 = 0$ and is called the irrotational solution. Therefore $\hat{v}$ can be resolved into two components and thus $\hat{v} = \hat{v}_R + \hat{v}_I$.

Now considering the case $K^2 \neq 0$, we assume the regular perturbation expansion of $\hat{v}$ in terms of the parameter N^2 in the form

$$\hat{v}_R(\alpha,\beta,\gamma,T) = \hat{v}_0(\alpha,\beta,\gamma,T) + N^2\hat{v}_1(\alpha,\beta,\gamma,T) + \left(N^2\right)^2\hat{v}_2(\alpha,\beta,\gamma,T) + \ldots \qquad (2.10)$$

where $\hat{v}_R$ is the rotational component of $\hat{v}$.

From the zeroth, first and second order,

$$\hat{v}_R = \frac{T\hat{\Omega}_0(\alpha,\beta,\gamma) + \hat{\Omega}_1(\alpha,\beta,\gamma)}{\bar{\alpha}^2 + (\beta-\sigma\alpha T)^2} - N^2\bar{\alpha}^2\left[\left(\frac{\beta\hat{\Omega}_0}{\sigma^2\alpha^2\bar{\alpha}} - \frac{\hat{\Omega}_1}{\sigma\alpha\bar{\alpha}}\right)\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right.$$

$$\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right) + \frac{\bar{\alpha}}{2\sigma\alpha}\log\left(\frac{\bar{\alpha}^2 + (\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right) + \frac{1}{2\sigma}\left(\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\log\left(\frac{\bar{\alpha}^2 + (\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)\right.$$

$$\left.\left.-2\left(\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right) - \tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)\right)\left(\frac{\hat{\Omega}_0}{\sigma^2\alpha^2\bar{\alpha}^2}\right)\right]\frac{1}{\bar{\alpha}^2 + (\beta-\sigma\alpha T)^2} + \left(N^2\right)^2\frac{\bar{\alpha}^3}{\sigma\alpha}\left[\left(\frac{\hat{\Omega}_0}{\sigma^2\alpha^2\bar{\alpha}} - \frac{\hat{\Omega}_1}{\sigma\alpha\bar{\alpha}}\right)\right.$$

$$\left[\frac{1}{2\sigma^2\alpha^2}\left(\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\log\left(\frac{\bar{\alpha}^2 + (\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)\right) - 2\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right.$$

$$-\frac{1}{2}\left(\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)\right)^2+\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)+\frac{(\beta-\sigma\alpha T)^4}{12\sigma\alpha\bar{\alpha}^2}\left(\frac{1}{\bar{\alpha}}+\frac{1}{\sigma\alpha\bar{\alpha}}\right)\Bigg]$$

$$-\frac{\hat{\Omega}_0}{8\sigma^4\alpha^4\bar{\alpha}}\left[\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\left(\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)\right)^2-2\left(\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)-\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)\right]$$

$$\left(\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)\right)-2\left(\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)-\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)\right)-\frac{1}{3}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^3\Bigg]$$

$$-\frac{\hat{\Omega}_0}{2\sigma^4\alpha^3\bar{\alpha}}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)-2\left(\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)-\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)+\frac{2}{\sigma^2\alpha^2}\Bigg[\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)$$

$$\cos\left(\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)-\sin\left(\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)\Bigg]\frac{1}{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2} \qquad (2.11)$$

The solution for $K^2 = 0$ which corresponds to irrotational motion is obtained by considering the two–dimensional Fourier transform of the perturbation equations instead of the full three–dimensional decomposition. Using moving co-ordinate transformation given by equation (2.6), $K^2\hat{v} = 0$ corresponds to

$$\frac{\partial^2\breve{v}_I}{\partial\eta^2}+2i\sigma\alpha T\frac{\partial\breve{v}_I}{\partial\eta}-\left(\bar{\alpha}^2+\sigma^2\alpha^2T^2\right)\breve{v}_I = 0, \qquad (2.12)$$

with

$$\breve{v}_I=\breve{v}_I(\alpha,\eta,\gamma;T) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} v_I(\xi,\eta,\varsigma,T)\,e^{i(\alpha\xi+\gamma\zeta)}\,d\xi\,d\zeta, \qquad (2.13)$$

is the irrotational part of $\hat{v}$. The solution of equation (2.14) is found to be

$$\breve{v}_I = A(T)e^{-\bar{\alpha}\eta-i\sigma\alpha T\eta}+B(T)e^{\bar{\alpha}\eta-i\sigma\alpha T\eta}, \qquad (2.14)$$

where A(T) and B(T) are constants of integration .

In order to combine $\hat{v}_R$ and $\breve{v}_I$ to obtain the complete the solution and satisfy the matching condition $\hat{v}_R$ must be inverted once to obtain $\breve{v}_R(\alpha,\eta,\gamma;T)$i,e.,

$$\breve{v}_R(\alpha,\eta,\gamma;T) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{v}_R(\alpha,\beta,\gamma;T)\,e^{-i\beta\eta}\,d\beta. \qquad (2.15)$$

With initial velocity and initial density as unit pulse, the initial conditions are expressed as

$$v(x,y,z,0)=V_0\,\delta(x-x_0)\delta(y-y_0)\delta(z-z_0). \qquad (2.16)$$

$$\rho(x,y,z,0)=\tilde{\rho}_0\,\delta(x-x_0)\delta(y-y_0)\delta(z-z_0). \qquad (2.17)$$

In terms of moving co-ordinates and three-dimensional Fourier transform ,equations (2.18) and (2.19) becomes

$$\breve{v}_0(\alpha,\beta,\gamma)=\Omega_0(\alpha,\beta,\gamma)=V_0 e^{i(\alpha x_0+\beta y_0+\gamma z_0)}. \tag{2.18}$$

$$\tilde{\rho}_0(\alpha,\beta,\gamma)=\Omega_1(\alpha,\beta,\gamma)=\tilde{\rho}_0 e^{i(\alpha x_0+\beta y_0+\gamma z_0)}. \tag{2.19}$$

$\breve{v}_R$ is found to be

$$\begin{aligned}
\breve{v}_R = {} & e^{i(\alpha x_0+\gamma z_0-\sigma\alpha T\bar{\eta})}\Big\{ e^{-\bar{\alpha}|\bar{\eta}|}\Big(TV_0+\tilde{\rho}_0+\frac{N^4\bar{\alpha}^2}{12\sigma^2\alpha^2}\Big(\frac{V_0}{\sigma^2\alpha^2\bar{\alpha}}-\frac{\tilde{\rho}_0}{\sigma\alpha\bar{\alpha}}\Big)\Big(1+\frac{1}{\sigma\alpha\bar{\alpha}}\Big)\Big) \\
& +\frac{e^{-2\bar{\alpha}\bar{\eta}}}{\bar{\eta}\alpha^2}\Big[\Big(\frac{V_0}{\sigma^2\alpha^2\bar{\alpha}}-\frac{\tilde{\rho}_0}{\sigma\alpha\bar{\alpha}}\Big)\big(N^2-2\bar{\alpha}^4N^4\big)-\frac{2i\,N^4\bar{\alpha}^3V_0}{\sigma^2\alpha^2\bar{\alpha}}-\frac{i\,N^4\bar{\alpha}^4V_0}{2\sigma^5\alpha^5}\Big] \\
& +\Big[\Big(\frac{V_0}{\sigma^2\alpha^2\bar{\alpha}}-\frac{\tilde{\rho}_0}{\sigma\alpha\bar{\alpha}}\Big)\Big(-N^2\alpha+\frac{2\bar{\alpha}^4N^4}{\sigma}\Big)+\frac{i\,N^4\bar{\alpha}V_0}{2\sigma^4\alpha^3\bar{\alpha}^2}\Big]\int_{-\infty}^{\infty}\frac{\eta' e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\bar{\eta}|}}{(\bar{\eta}-\eta')}d\eta' \\
& +\Big(\frac{V_0}{\sigma^2\alpha^2\bar{\alpha}}-\frac{\tilde{\rho}_0}{\sigma\alpha\bar{\alpha}}\Big)\Big(\frac{N^2}{2\sigma}\Big)\int_{-\infty}^{\infty}\frac{(\bar{\eta}-\eta')e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\bar{\eta}|}}{\eta'}d\eta'-iV_0\bar{\eta}e^{-\bar{\alpha}|\bar{\eta}|} \\
& \Big(\frac{N^2}{\sigma^2\alpha^2\bar{\alpha}^2}-\frac{11\bar{\alpha}^3N^4}{48\sigma^5\alpha^5}-\frac{\bar{\alpha}N^4}{2\sigma^4\alpha^3\bar{\alpha}^2}\Big)-\Big(\frac{V_0}{\sigma^2\alpha^2\bar{\alpha}}-\frac{\tilde{\rho}_0}{\sigma\alpha\bar{\alpha}}\Big)\Big(\Big(\frac{N^4\bar{\alpha}^3}{2\sigma^3\alpha^3}-\frac{N^4\bar{\alpha}^3}{2\sigma\alpha}\Big) \\
& -\Big(\frac{V_0N^4}{8\sigma^4\alpha^4\bar{\alpha}}-\frac{2iV_0\bar{\alpha}^2N^4}{8\sigma^5\alpha^5}\Big)\Big)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-\bar{\alpha}|\bar{\eta}-\eta'-\eta''|-\bar{\alpha}|\eta'-\eta''|-\bar{\alpha}|\eta''|}}{(\eta'-\eta'')\,\eta''}d\eta'd\eta'' \\
& +\Big(\frac{iV_0N^4}{\sigma^4\alpha^3\bar{\alpha}}\Big(1+\frac{1}{\sigma^2\alpha^2}\Big)\Big)\int_{-\infty}^{\infty}\frac{e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}}{(\bar{\eta}-\eta')}d\eta'\Big\}.
\end{aligned} \tag{2.20}$$

Here $\bar{\eta}=\eta-y_0$. Now the complete solution will be

$$\breve{v}=\breve{v}_R+\breve{v}_I. \tag{2.21}$$

$\breve{v}_R$ and $\breve{v}_I$ given by equations (2.20) and (2.14).

III. STRATIFIED BOUNDED COUETTE FLOW

In this case, a stratified plane Couette flow bounded at $y=\pm H$ is considered (Fig. 1). Here velocity $\breve{v}$ vanishes at $\eta=\pm H$, hence we have

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$$e^{-\bar{\alpha}H-i\sigma\alpha TH}A+e^{\bar{\alpha}H-i\sigma\alpha TH}B=-\left[\breve{v}_R\right]_{\eta=+H}, \tag{3.1}$$

$$e^{\bar{\alpha}H+i\sigma\alpha TH}A+e^{-\bar{\alpha}H+i\sigma\alpha TH}B=-\left[\breve{v}_R\right]_{\eta=-H}, \tag{3.2}$$

From equations(3.1) and (3.2), A and B are found to be

$$A=\frac{\breve{v}_R(+H)e^{-\bar{\alpha}H+i\sigma\alpha TH}-\breve{v}_R(+H)e^{\bar{\alpha}H-i\sigma\alpha TH}}{2\sinh(2\bar{\alpha}H)}. \tag{3.3}$$

$$B=\frac{\breve{v}_R(-H)e^{-\bar{\alpha}H-i\sigma\alpha TH}-\breve{v}_R(-H)e^{\bar{\alpha}H+i\sigma\alpha TH}}{2\sinh(2\bar{\alpha}H)}. \tag{3.4}$$

where $\breve{v}_R(\pm H)=-\left[\breve{v}_R\right]_{\eta=\pm H}$.

It is found that

$$\breve{v}_R(+H)=\left(A_1T+B_1\right)e^{i\left(\alpha x_0+\gamma z_0-\sigma\alpha T\left(H-y_0\right)\right)}. \tag{3.5}$$

$$\breve{v}_R(-H)=\left(A_2T+B_2\right)e^{i\left(\alpha x_0+\gamma z_0+\sigma\alpha T\left(H+y_0\right)\right)}. \tag{3.6}$$

The values of the coefficients are given in **APPENDIX .**

IV. RESULTS AND DISCUSSIONS

In this problem, we have studied the evolution of linearized disturbances of a basic flow of an inviscid stratified bounded couette flow with unit pulse for velocity and density as initial distributions. Here, we have resolved the disturbances into rotational and irrotational components.

Figs. 2(a)–(b) are plots of $|\hat{\rho}|$ versus T for different values of Brunt Väisälä frequency N (N = 0, 0.2, 0.5) and φ ($\varphi = 0^0, 45^0$) and figs. 3(a)–(b) are plots of $|\hat{\rho}|$ versus T for different values of Brunt Väisälä frequency N (N = 0, 0.2, 0.5) and φ ($\varphi = 0^0, 45^0$). It is observed that as the value of N increases $\hat{v}_R$ decays at a faster rate for large time. Hence we can conclude that stratification stabilizes the flow velocity but there is growth in the perturbation density.

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APPENDIX

$$A_1 = V_0\, e^{-\bar{\alpha}\left|H - y_0\right|},\; A_2 = V_0\, e^{-\bar{\alpha}\left|-H - y_0\right|},\; B_2 = B_1\Big|_{H=-H}$$

$$B_1 = e^{-\bar{\alpha}\left|H-y_0\right|}\left(\tilde{\rho}_0 + \frac{N^4\bar{\alpha}^2}{12\sigma^2\alpha^2}\left(\frac{V_0}{\sigma^2\alpha^2\bar{\alpha}} - \frac{\tilde{\rho}_0}{\sigma\alpha\bar{\alpha}}\right)\left(1+\frac{1}{\sigma\alpha\bar{\alpha}}\right)\right) - \frac{e^{-2\bar{\alpha}\left(H-y_0\right)}}{\left(H-y_0\right)\bar{\alpha}^2}\left[\left(\frac{V_0}{\sigma^2\alpha^2\bar{\alpha}} - \frac{\tilde{\rho}_0}{\sigma\alpha\bar{\alpha}}\right)\right.$$

$$\left.\left(N^2 - 2\bar{\alpha}^4 N^4\right) - \frac{2i\, N^4\bar{\alpha}^3 V_0}{\sigma^2\alpha^2\bar{\alpha}} - \frac{i\, N^4\bar{\alpha}^4 V_0}{2\sigma^5\alpha^5}\right] - \left(\frac{V_0}{\sigma^2\alpha^2\bar{\alpha}} - \frac{\tilde{\rho}_0}{\sigma\alpha\bar{\alpha}}\right)\left(\left(-N^2\alpha + \frac{2\bar{\alpha}^4 N^4}{\sigma}\right)\right.$$

$$\left.\int_{-\infty}^{\infty} \frac{\eta' e^{-\bar{\alpha}\left|H-y_0-\eta'\right| - \bar{\alpha}\left|\eta'\right|}}{\left(H-y_0-\eta'\right)}\, d\eta' + \left(\frac{N^2}{2\sigma}\right)\int_{-\infty}^{\infty} \frac{\left(H-y_0-\eta'\right) e^{-\bar{\alpha}\left|H-y_0-\eta'\right| - \bar{\alpha}\left|\eta'\right|}}{\eta'}\, d\eta'\right)$$

$$-iV_0\left(\frac{N^2}{\sigma^2\alpha^2\bar{\alpha}^2} - \frac{11\bar{\alpha}^3 N^4}{48\sigma^5\alpha^5} - \frac{\bar{\alpha}\, N^4}{2\sigma^4\alpha^3\bar{\alpha}^2}\right)\left(H-y_0\right)e^{-\bar{\alpha}\left|H-y_0\right|} - \left(\frac{V_0}{\sigma^2\alpha^2\bar{\alpha}} - \frac{\tilde{\rho}_0}{\sigma\alpha\bar{\alpha}}\right)$$

$$\left(\frac{N^4\bar{\alpha}^3}{2\sigma^3\alpha^3} - \frac{N^4\bar{\alpha}^3}{2\sigma\,\alpha}\right)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{e^{-\bar{\alpha}\left|H-y_0-\eta'-\eta''\right| - \bar{\alpha}\left|\eta'-\eta''\right| - \bar{\alpha}\left|\eta''\right|}}{\left(\eta'-\eta''\right)\eta''}\, d\eta' d\eta'' +$$

$$\left(\frac{V_0 N^4}{8\sigma^4\alpha^4\bar{\alpha}} - \frac{2iV_0\bar{\alpha}^2 N^4}{8\sigma^5\alpha^5}\right)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{e^{-\bar{\alpha}\left|H-y_0-\eta'-\eta''\right| - \bar{\alpha}\left|\eta'-\eta''\right| - \bar{\alpha}\left|\eta''\right|}}{\left(H-y_0-\eta'-\eta''\right)\left(\eta'-\eta''\right)\eta''}\, d\eta' d\eta''$$

$$\left. + \left(\frac{iV_0 N^4}{\sigma^4\alpha^3\bar{\alpha}}\left(1+\frac{1}{\sigma^2\alpha^2}\right)\right)\int_{-\infty}^{\infty} \frac{e^{-\bar{\alpha}\left|H-y_0-\eta'\right| - \bar{\alpha}\left|\eta'\right|}}{\left(H-y_0-\eta'\right)}\, d\eta'\right\}$$

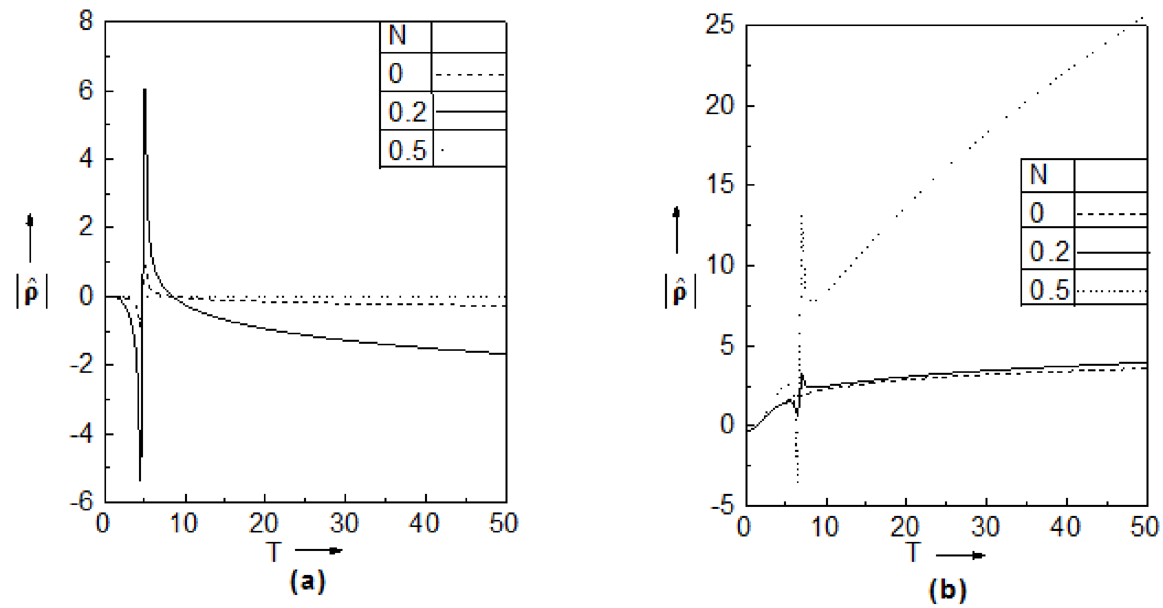

Fig. 2 Plots of $|\hat{\rho}|$ *versus T for (a)* $\varphi = 0^0$ *and (b)* $\varphi = 45^0$*, for different values of N.*

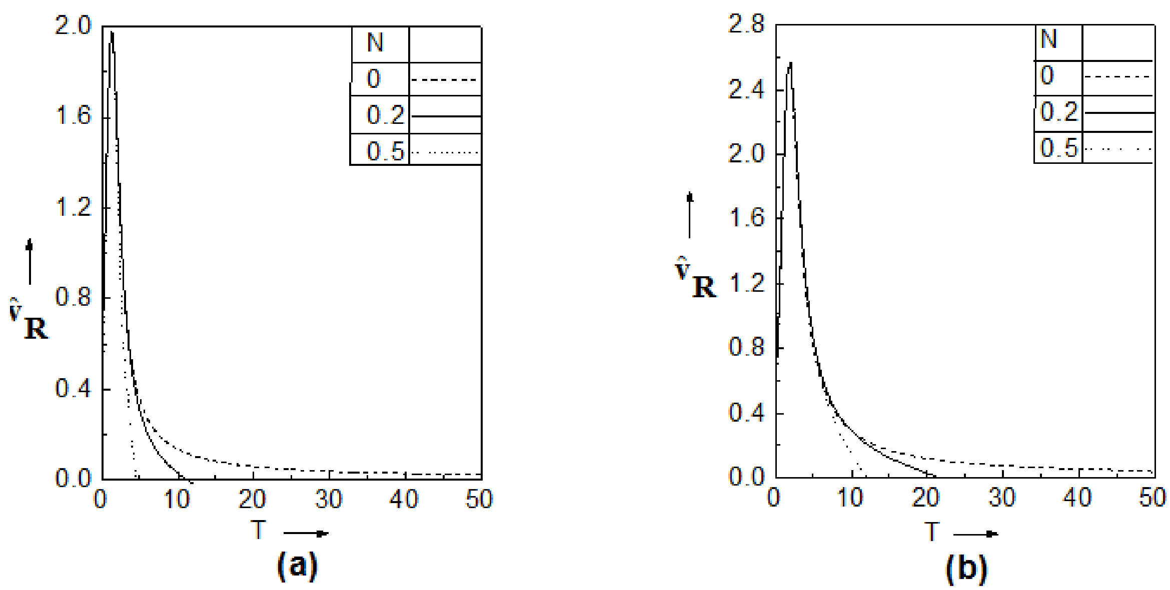

Fig. 3 Plots of $\hat{v}_R$ *versus T for (a)* $\varphi = 0^0$ *and (b)* $\varphi = 45^0$*, for different values of N.*

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